

# Complete Hamiltonian analysis of cosmological perturbations at all orders II: Non-canonical scalar field

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**Abstract.** In this work, we present a consistent Hamiltonian analysis of cosmological perturbations for generalized non-canonical scalar fields. In order to do so, we introduce a new phase-space variable that is uniquely defined for different non-canonical scalar fields. We also show that this is the simplest and efficient way of expressing the Hamiltonian. We extend the Hamiltonian approach of [1] to non-canonical scalar field and obtain an unique expression of speed of sound in terms of phase-space variable. In order to invert generalized phase-space Hamilton's equations to Euler-Lagrange equations of motion, we prescribe a general inversion formulae and show that our approach for non-canonical scalar field is consistent. We also obtain the third and fourth order interaction Hamiltonian for generalized non-canonical scalar fields and briefly discuss the extension of our method to generalized Galilean scalar fields.

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## 1 Introduction

The inflationary paradigm is highly successful and an attractive way of resolving some of the puzzles of standard cosmology. During inflation, the early universe undergoes an accelerated expansion, stretching quantum fluctuations to super-horizon scales which we observe today as CMB anisotropy [2]. Since Einstein's equations are highly non-linear, comparison of the predictions of inflation with the observations require one to expand the equations order-by-order. At linear order, predictions of inflation is consistent with CMB. However, the linear order observables, like scalar spectral index and tensor-to-scalar ratio, can not rule out models of inflation; physically measurable observables corresponding to higher-order quantities like bispectrum or trispectrum will help to rule out some models of inflation [3–6].

In the standard inflationary models, inflation is driven by scalar field(s). The canonical scalar fields are the simplest and to get sufficient amount of inflation require flat-potential. During canonical 'slow-roll' evolution, potential energy dominates over its kinetic energy and drives a quasi-exponential expansion of the universe which is often difficult to obtain within particle physics models [3, 4]. Non-canonical scalar fields are generalizations of canonical scalar fields and reduces the dependence on the potential. In case of non-canonical scalar field, even in the absence of potential energy term, a general class of non-quadratic kinetic

terms can drive inflationary evolution. This model satisfy two crucial requirements of inflationary scenarios: the scalar perturbations are well-behaved during inflation and there exists a natural mechanism for exiting inflation in a graceful manner. Non-canonical scalar field also contains extra parameters than canonical scalar field such as speed of sound. However, unlike canonical scalar field models, the speed of propagation of the scalar perturbations in these inflationary models can be time-dependent[7–9]. Recently, in order to seek more generalized field, scalar fields with higher time derivatives models like Hordenski scalar fields, Kinetic Gravity Braiding models [10–15] are considered. Besides these, there are plenty of other models that lead to accelerated universe.

Since inflation takes place at high-energies, quantum field theory is the best description of the matter at these energies. Hence, evaluation of any physical quantity, like the  $n$ -point correlation functions, require us to either promote effective field variables (using Heisenberg picture) to operators or integrate over all possible field configurations on all of space-time (path - integral picture). Since it is unclear what effective field configurations to integrate over, Heisenberg picture is the preferred approach. In other words, we obtain the effective Hamiltonian operator and evaluate the correlation functions of the relevant operator.

In the context of cosmological perturbation theory, there are currently two approaches in the literature to evaluate the effective Hamiltonian — Lagrangian formalism and Hamiltonian formalism. In case of Lagrangian formalism, the Lagrangian is expanded upto a particular order, i.e., if we are interested in obtaining third order interaction Hamiltonian, effective Lagrangian needs to be expanded up to third order and constraints are systematically removed from the system to obtain the effective perturbed Lagrangian. Then, the momentum  $\pi$  corresponding to  $\varphi$  is obtained as a polynomial of  $\dot{\varphi}$  and using order-by-order approximations,  $\dot{\varphi}$  is expressed as a polynomial of  $\pi$ . Next, using Legendre transformation, Hamiltonian is expressed in terms of  $\pi$  and  $\varphi$ . In order to express the Hamiltonian in terms of  $\dot{\varphi}$  and  $\varphi$ , only the leading order relation between  $\pi$  and  $\dot{\varphi}$  is used [16–23]. There are some difficulties with the previous method:

1. In case of cosmological perturbations,  $\pi$  and  $\dot{\varphi}$  are perturbed quantities (curvature perturbation), expressing one in terms of polynomial of other is an approximation.
2. At the end, to express the Hamiltonian in terms of configuration-space variable, we use only the leading order relation between  $\pi$  and  $\dot{\varphi}$ , not the polynomial relation. Hence, several approximations are employed to convert effective Lagrangian to effective Hamiltonian.
3. The above method is also very restrictive and it is difficult to extend the method for a generalized constrained system.
4. Also, it is difficult to use this method for higher order perturbations.

In the context of cosmological perturbations, the above approach leads to consistent results, however, a consistent Hamiltonian formulation is always preferred than the previous approach to make calculations simpler with more technical details. Langolois [24] first introduced a consistent Hamiltonian formulation of canonical scalar field. However, Langolois’ approach is also difficult to extend to higher order of perturbations or to any different types of field due to the fact that it requires construction of gauge-invariant conjugate momentum. In our recent work [1], henceforth referred as I, we have introduced a different Hamiltonian approach that can address and deal with all the issues in the previous methods and provides

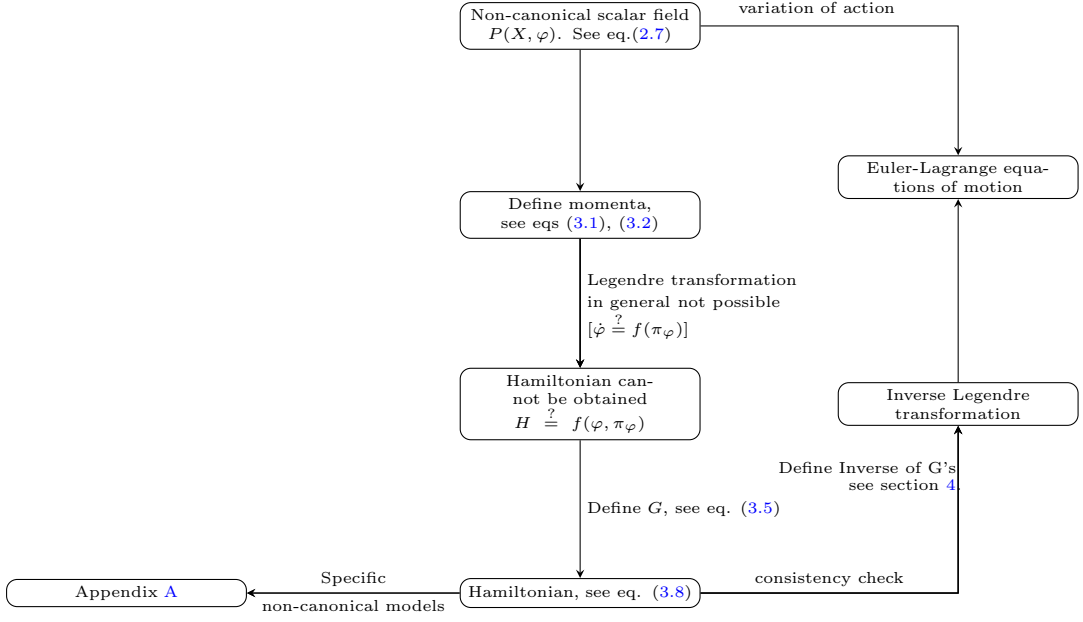
an effective and robust way to obtain interaction Hamiltonian for any model for any order of perturbations. Also, in case of calculating mixed-mode (e.g., scalar-tensor) interaction Hamiltonian [22, 23], our approach is simpler than the previous one. Table below provides a bird's eye view of the both the formulations and advantages of the Hamiltonian formulation that is proposed in this work [1]

	Lagrangian formulation	Hamiltonian formulation
Gauge conditions and gauge-invariant equations	At any order, choose a gauge which does not lead to gauge-artifacts	Choose a gauge with no gauge-artifacts, however, momentum corresponding to unperturbed quantity is non-zero leading to consistent equations of motion.
Dynamical variables	Counting true dynamical degrees of freedom is difficult.	Using Dirac's procedure, constraints can easily be obtained and is easy to determine the degrees of freedom.
Quantization at all orders	Difficult to quantize constrained systems.	Since constraints are obtained systematically and reduced phase space contains only true degrees of freedom, it is straightforward to quantize the theory using Hamiltonian formulation.
Calculating the observables	Requires to invert the expressions at each order and hence non-trivial to compute higher-order correlation function.	Once the relation between $\varphi$ and Curvature perturbation <sup>1</sup> is known, calculating the correlation functions from the Hamiltonian is simple and straightforward to obtain.

In I, we applied our new method to canonical and a specific higher derivative (Galilean) scalar field model, and showed explicitly that the method can efficiently obtain Hamiltonian at all orders. In the case of certain non-canonical scalar field models, if  $\dot{\varphi}$  can be expressed uniquely in terms of the canonical conjugate momentum, it is then possible to obtain Hamiltonian and the results of I can be extended. However, for a general non-canonical scalar field, it is not possible to do the procedure as we do not have a way to rewrite  $\dot{\varphi}$  in terms of the canonical conjugate momentum and, hence, it is not possible to obtain the Hamiltonian for general non-canonical scalar field. In this work, we explicitly obtain Hamiltonian for a general non-canonical scalar fields and obtain interaction Hamiltonian upto fourth order.

This work is divided into two parts. In the first part, we provide the procedure to obtain Hamiltonian for the non-canonical scalar field by introducing a phase-space variable. Then, by choosing different models, we explicitly show that the Hamiltonian leads to consistent equations of motion as well as perturbed interaction Hamiltonian by implementing our approach. We also find a new definition of speed of sound in terms of phase-space variables. In the second part, in order to retrieve generalized equations of motion in configuration-space from phase-space, we provide a systematic way to invert generalized non-canonical phase-space variables to configuration-space variables and vice versa and show that, all equations are consistent. Finally, we extend the method to generalized higher derivative scalar fields. A flow-chart below illustrates the method for non-canonical scalar fields:

<sup>1</sup>It is important to note that, in the case of first order, relation between  $\varphi$  and three-curvature is straightforward. However, it is more subtle in the case of higher-order perturbations[25].



In the next section, we briefly discuss about non-canonical scalar fields. We also discuss the gauge choices and corresponding gauge-invariant variables. In section 3, Hamiltonian formulation of generalized non-canonical scalar field is introduced by defining a new phase-space function which provides consistent equations of motion. In section 3.2, we extend the results of I to non-canonical scalar field in flat-slicing gauge to obtain perturbed equations of motion. In section 4, we provide a partial inversion method between phase-space variables and configuration-space variables and in section 5, we provide the third and fourth order Interaction Hamiltonian for non-canonical scalar field. In section 6, we briefly discuss the application of our method to generalized Galilean scalar field model. Finally, in section 7, we end with discussions and conclusions of the results. In Appendix A, functional form of the new variable is obtained for different scalar field models. In Appendix B, we implement Langlois' approach to non-canonical scalar field model.

In this work, we consider  $(-, +, +, +)$  metric signature. We also denote  $\prime$  as derivative with respect to conformal time.

## 2 Model and gauge choices

Action for non-canonical scalar field minimally coupled to gravity is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + \mathcal{L}_m(\varphi, \partial\varphi) \right], \quad (2.1)$$

where  $R$  is the Ricci scalar and the matter Lagrangian,  $\mathcal{L}_m$  is of the form.

$$\mathcal{L}_m = P(X, \varphi), \quad X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi. \quad (2.2)$$

$P(X, \varphi)$  corresponds to non-standard kinetic term and hence the name non-canonical scalar field[7–9]. Further, fixing  $P = -X - V(\varphi)$ , where  $V(\varphi)$  is the potential, one can

retrieve the well-known canonical scalar field model. Varying the action (2.1) with respect to metric gives Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R = \kappa T_{\mu\nu}, \quad (2.3)$$

where the stress tensor ( $T_{\mu\nu}$ ) for non-canonical scalar field is

$$T_{\mu\nu} = -P_X \partial_\mu \varphi \partial_\nu \varphi + g_{\mu\nu} P. \quad (2.4)$$

Varying the action (2.1) with respect to the scalar field ' $\varphi$ ' leads to the following equation of motion

$$P_X \square \varphi - P_{XX} \partial_\mu \varphi \partial^\mu \varphi - 2X P_{X\varphi} + P_\varphi = 0 \quad (2.5)$$

which can also be obtained from the conservation of Energy-Momentum tensor,  $\nabla_\mu T^{\mu\nu} = 0$ .

The four-dimensional line element in the ADM form is given by,

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -(N^2 - N_i N^i) d\eta^2 + 2N_i dx^i d\eta + \gamma_{ij} dx^i dx^j, \end{aligned} \quad (2.6)$$

where  $N(x^\mu)$  and  $N_i(x^\mu)$  are Lapse function and Shift vector respectively,  $\gamma_{ij}$  is the 3-D space metric. Action (2.1) for the line element (2.6) takes the form

$$\mathcal{S}_{NC} = \int d^4x N \sqrt{\gamma} \left[ \frac{1}{2\kappa} \left( {}^{(3)}R + K_{ij} K^{ij} - K^2 \right) + P(X, \varphi) \right] \quad (2.7)$$

where  $K_{ij}$  is extrinsic curvature tensor and is defined by

$$\begin{aligned} K_{ij} &\equiv \frac{1}{2N} (\partial_0 \gamma_{ij} - N_{i|j} - N_{j|i}), \\ K &\equiv \gamma^{ij} K_{ij}. \end{aligned}$$

Perturbatively expanding the metric only in terms of scalar perturbations and the scalar field about the flat FRW spacetime in conformal coordinate, we get,

$$g_{00} = -a(\eta)^2 (1 + 2\epsilon\phi_1 + \epsilon^2\phi_2 + \dots) \quad (2.8)$$

$$g_{0i} \equiv N_i = a(\eta)^2 (\epsilon\partial_i B_1 + \frac{1}{2}\epsilon^2\partial_i B_2 + \dots) \quad (2.9)$$

$$g_{ij} = a(\eta)^2 ((1 - 2\epsilon\psi_1 - \epsilon^2\psi_2 - \dots)\delta^{ij} + 2\epsilon E_{1ij} + \epsilon^2 E_{2ij} + \dots) \quad (2.10)$$

$$\varphi = \varphi_0(\eta) + \epsilon\varphi_1 + \frac{1}{2}\epsilon^2\varphi_2 + \dots \quad (2.11)$$

where  $\epsilon$  denotes the order of the perturbation. To determine the dynamics at every order, we need five scalar functions ( $\phi, B, \psi, E$  and  $\varphi$ ) at each order. Since there are two arbitrary gauge-freedoms for scalar perturbations, one can fix two of the five scalar functions. In this work, we derive all equations by choosing a specific gauge — flat-slicing gauge, i.e.,  $\psi = 0, E = 0$  — at all orders:

$$g_{00} = -a(\eta)^2 (1 + 2\epsilon\phi_1 + \epsilon^2\phi_2 + \dots) \quad (2.12)$$

$$g_{0i} \equiv N_i = a(\eta)^2 (\epsilon\partial_i B_1 + \frac{1}{2}\epsilon^2\partial_i B_2 + \dots) \quad (2.13)$$

$$g_{ij} = a(\eta)^2 \delta_{ij} \quad (2.14)$$

$$\varphi = \varphi_0(\eta) + \epsilon\varphi_1 + \frac{1}{2}\epsilon^2\varphi_2 + \dots \quad (2.15)$$

It can be shown that, perturbed equations in flat-slicing gauge coincide with gauge-invariant equations of motion (in generic gauge,  $\varphi_1$  coincides with  $\varphi_1 + \frac{\varphi_0'}{H}\psi_1 \equiv \frac{\varphi_0'}{H}\mathcal{R}$  which is a gauge-invariant quantity,  $\mathcal{R}$  is called curvature perturbation [25]). Similarly, one can choose another suitable gauge with no coordinate artifacts to obtain gauge-invariant equations of motion. Such gauges are Newtonian-conformal gauge ( $B = 0$ ,  $E = 0$ ), constant density gauge ( $E = 0$ ,  $\delta\varphi = 0$ ), etc.

Before we proceed with the Hamiltonian formulation, it is important to clarify issues related to the quantization in the cosmological perturbation theory: While the field variable  $\varphi_i$  and metric variables  $\phi_i, B_i, \psi_i$  (where  $i$  takes values  $1, 2, \dots$ ) are expanded perturbatively, it is important to note that the operators corresponding to these variables (i. e.  $\varphi_1, \varphi_2, \dots$ ) can not be treated as independent operators as higher orders in perturbation theory do not lead to independent degrees of freedom. As otherwise, the unperturbed theory should have infinitely many local degrees of freedom. In a canonical quantization, there is one operator and one for its momentum, on which the quantum Hamiltonian depend<sup>1</sup>.

### 3 Hamiltonian formulation

In our last work [1], we provided an efficient way of obtaining consistent perturbed Hamiltonian for any gravity models. However, it works only if the form of the action is specified in such a way that the Legendre transformation  $\dot{\varphi} \rightarrow \pi_\varphi$  is invertible in both ways. For non-canonical scalar fields, momenta corresponding to the action (2.7) are

$$\pi^{ij} \equiv \frac{\delta \mathcal{S}_{NC}}{\delta \gamma'_{ij}} = \frac{\sqrt{\gamma}}{2\kappa} (\gamma^{ij}\gamma^{kl} - \gamma^{ik}\gamma^{jl})K_{kl} \quad (3.1)$$

$$\pi_\varphi \equiv \frac{\delta \mathcal{S}_{NC}}{\delta \varphi'} = -\sqrt{\gamma}P_X\sqrt{-2X+Y}, \quad \text{where} \quad Y \equiv \gamma^{ij}\partial_i\varphi\partial_j\varphi. \quad (3.2)$$

As one can see, equation (3.1) is invertible and the inversion relation is given by

$$\gamma'_{mn} = \gamma_{nk}N^k_{|m} + \gamma_{mk}N^k_{|n} - 2NK_{mn}, \quad K_{ij} = \frac{\kappa}{\sqrt{\gamma}} (\gamma_{ij}\gamma_{kl} - 2\gamma_{ik}\gamma_{jl})\pi^{kl} \quad (3.3)$$

but equation (3.2) is non-invertible for arbitrary function of  $P(X, \varphi)$ . However, if  $P(X, \varphi)$  is specified, it may be possible to invert the equation and  $X$  can be written in terms of  $\pi_\varphi$ . Inversion relations for commonly used non-canonical models are given in Appendix A. Using equation (3.3), we can write the Hamiltonian density as

$$\begin{aligned} \mathcal{H}_{NC} &= \pi^{ij}\gamma'_{ij} + \pi_\varphi\varphi' - \mathcal{L}_{NC} \\ &= 2\gamma_{ij}\partial_k N^j \pi^{ik} + N^i\partial_i\gamma_{jk} \pi^{jk} - \frac{N\kappa}{\sqrt{\gamma}} (\gamma_{ij}\gamma_{kl} - 2\gamma_{ik}\gamma_{jl})\pi^{ij}\pi^{kl} - \frac{N\sqrt{\gamma}}{2\kappa} {}^{(3)}R - \\ &\quad N\sqrt{\gamma} \tilde{G}(X, \gamma, Y, \varphi) + N^i\pi_\varphi\partial_i\varphi, \quad \text{where} \quad \tilde{G} \equiv (P - P_X(2X - Y)). \end{aligned} \quad (3.4)$$

Note that, the above expression is still not the Hamiltonian since  $\tilde{G}$  is not a phase-space variable and it is not invertible for arbitrary form of  $P(X, \varphi)$  since equation (3.2) is not invertible, in general. Hence, a natural question that arises is: *How to invert configuration-space variables to phase-space variables so that we can obtain generalized consistent Hamiltonian for non-canonical scalar field?*

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<sup>1</sup>We thank Martin Bojowald for discussion regarding this point.

In this section, we show that, by defining a new phase-space function, the above problem can be resolved. The new phase-space quantity is defined as

$$G(\pi_\varphi, \gamma, Y, \varphi) = \tilde{G}(X, \gamma, Y, \varphi) \equiv P - P_X (2X - Y). \quad (3.5)$$

Since momenta corresponding to  $N$  and  $N^i$  vanish, i.e.,  $\pi_N = \pi_i = 0$ , using the above defined variable, Hamiltonian constraint can be written as

$$\mathcal{H}_N \equiv \{\pi_N, \mathcal{H}_{NC}\} = \frac{\delta \mathcal{H}_{NC}}{\delta N} = -\frac{\kappa}{\sqrt{\gamma}} (\gamma_{ij} \gamma_{kl} - 2\gamma_{ik} \gamma_{jl}) \pi^{ij} \pi^{kl} - \frac{\sqrt{\gamma}}{2\kappa} {}^{(3)}R - \sqrt{\gamma} G(\pi_\varphi, \gamma, Y, \varphi) = 0, \quad (3.6)$$

and Momentum constraint is given by

$$M_i \equiv \{\pi_i, \mathcal{H}_{NC}\} = \frac{\delta \mathcal{H}_{NC}}{\delta N^i} = -2\partial (\gamma_{im} \pi^{mn}) + \pi^{kl} \partial_i \gamma_{kl} + \pi_\varphi \partial_i \varphi = 0. \quad (3.7)$$

Due to diffeomorphic invariance, total Hamiltonian density can be written as

$$\mathcal{H}_{NC} = N \mathcal{H}_N + N^i \mathcal{H}_i = 0. \quad (3.8)$$

Instead of defining  $G$ , one can define any other phase-space variable(s) and express the Hamiltonian in a different form and the possibilities are infinite. However, as one can see, since  $\tilde{G}(X, \gamma, Y, \varphi)$  automatically appears directly in the Hamiltonian, this is the simplest and effective way to express the Hamiltonian for non-canonical scalar field.  $G$  not only resolves the issue of expressing Hamiltonian for non-canonical scalar field and is also uniquely defined for different non-canonical scalar fields. Hence,  $G$  carries the signature of the non-canonical scalar fields in phase-space. Explicit forms of  $G(\pi_\varphi, \gamma, Y, \varphi)$  for different types of scalar fields are given in Appendix A.

### 3.1 Zeroth order

At zeroth order, since  $\gamma_{ij} = a^2 \delta_{ij}$  and all quantities are independent of spatial coordinates, we then get

$$\pi_0^{ij} = \frac{1}{6a} \pi_a \delta^{ij} \quad (3.9)$$

and Hamiltonian density at zeroth order becomes

$$\mathcal{H}_0 = -\frac{N_0 \kappa}{12a} \pi_a^2 - G N_0 a^3. \quad (3.10)$$

Variation of the Hamiltonian (3.10) with respect to the momenta leads to two equations and are given by

$$a' = -\frac{N_0 \kappa}{6a} \pi_a \quad (3.11)$$

$$\varphi'_0 = -N_0 a^3 G_{\pi_\varphi}. \quad (3.12)$$

Hamiltonian constraint leads to the equation of motion of  $N$  and at zeroth order, it is given by

$$\mathcal{H}_{N0} \equiv -\frac{\kappa}{12a} \pi_a^2 - G a^3 = 0. \quad (3.13)$$



Equations of motion are obtained by varying the Hamiltonian (3.10) with respect to field variables. Hence, equation of motion of  $a$  is obtained by the relation

$$\pi'_a = -\frac{\delta \mathcal{H}_0}{\delta a} = -\frac{N_0 \kappa}{12a^2} \pi_a^2 + 3G N_0 a^2 + G_a N_0 a^3. \quad (3.14)$$

Similarly, equation of motion of  $\varphi_0$  can be obtained from

$$\pi'_{\varphi 0} = N_0 a^3 G_\varphi. \quad (3.15)$$

### 3.2 First order

As we have mentioned in the introduction, there are two ways to obtain Hamiltonian — Langlois' approach [24], and the approach used in I [1]. In this work, we use both the approaches and explicitly show that it is possible to obtain a consistent Hamiltonian for non-canonical scalar fields. In Appendix B, we extend Langlois' approach to non-canonical scalar field and in the rest of the section, we extend I to obtain a consistent Hamiltonian for non-canonical scalar field.

The field variables and their corresponding momenta can be separated into unperturbed and perturbed parts as

$$N = N_0 + \epsilon N_1, \quad N^i = \epsilon N_1^i, \quad \varphi = \varphi_0 + \epsilon \varphi_1 \quad (3.16)$$

$$\pi^{ij} = \pi_0^{ij} + \epsilon \pi_1^{ij}, \quad \pi_\varphi = \pi_{\varphi 0} + \epsilon \pi_{\varphi 1} \quad (3.17)$$

and by using Taylor expansion of phase-space variable  $G(\pi_\varphi, \gamma, Y, \varphi)$ , the second order perturbed Hamiltonian density is given by

$$\begin{aligned} \mathcal{H}_2 = & \delta_{ij} \partial_k N_1^j (\pi_1^{ik} + \pi_1^{ki}) a^2 - N_0 \kappa a (\delta_{ij} \delta_{kl} - 2\delta_{ik} \delta_{jl}) \pi_1^{ij} \pi_1^{kl} - 2N_1 \kappa a (\delta_{ij} \delta_{kl} - 2\delta_{ik} \delta_{jl}) \pi_0^{ij} \pi_1^{kl} \\ & - G_\varphi N_1 a^3 \varphi_1 - G_{\pi_\varphi} N_1 \pi_{\varphi 1} a^3 - \frac{1}{2} G_{\varphi\varphi} N_0 \varphi_1^2 a^3 - \frac{1}{2} G_{\pi_\varphi \pi_\varphi} N_0 \pi_{\varphi 1}^2 a^3 - G_{\varphi \pi_\varphi} N_0 \pi_{\varphi 1} a^3 \varphi_1 \\ & - G_Y N_0 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a + N_1^i \pi_{0\varphi} \partial_i \varphi_1. \end{aligned} \quad (3.18)$$

Note that, as we have pointed out in I [1], perturbed momentum corresponding to an unperturbed variable may arise due to the presence of other perturbed phase-space variables, thus  $\pi_1^{ij}$  is non-zero and can be obtained by varying the Hamiltonian (3.18) with respect to  $\pi_1^{ij}$ :

$$\frac{\delta \mathcal{H}_2}{\delta \pi_1^{ij}} = 0 \Rightarrow \pi_1^{ij} = \frac{a}{2N_0 \kappa} \delta^{ij} \partial_k N_1^k - \frac{a}{4N_0 \kappa} \delta^{ki} \partial_k N_1^j - \frac{a}{4N_0 \kappa} \delta^{kj} \partial_k N_1^i - \frac{N_1}{N_0} \pi_0^{ij}. \quad (3.19)$$

Varying the perturbed Hamiltonian (3.18) with respect to  $\pi_{\varphi 1}$  leads to the following equation

$$\varphi'_1 = -G_{\pi_\varphi} N_1 a^3 - G_{\pi_\varphi \pi_\varphi} N_0 \pi_{\varphi 1} a^3 - G_{\varphi \pi_\varphi} N_0 \varphi_1 a^3 \quad (3.20)$$

$$\Rightarrow \pi_{\varphi 1} = -\frac{1}{N_0 a^3 G_{\pi_\varphi \pi_\varphi}} (\varphi'_1 + G_{\pi_\varphi} N_1 a^3 + G_{\varphi \pi_\varphi} N_0 \varphi_1 a^3). \quad (3.21)$$

Hamiltonian constraint is obtained by varying the Hamiltonian with respect to Lapse function. Hence, varying (3.18) with respect to  $N_1$  leads to first order Hamiltonian constraint and takes the form

$$-2\delta_{ij} \delta_{kl} \pi_0^{ij} \pi_1^{kl} + 4\delta_{ij} \delta_{kl} \pi_0^{ik} \pi_1^{jl} - G_\varphi a^2 \varphi_1 - G_{\pi_\varphi} \pi_{\varphi 1} a^2 = 0. \quad (3.22)$$

Similarly, by varying Hamiltonian with respect to  $N_1^i$  we get the following Momentum constraint,

$$\pi_{\varphi 0} \partial_i \varphi_1 - 2a^2 \delta_{ij} \partial_k \pi_1^{kj} = 0. \quad (3.23)$$

Finally, equation of motion of  $\varphi_1$  is obtained by varying the Hamiltonian with respect to  $\varphi_1$ , i.e.,

$$\pi'_{\varphi 1} = a^3 G_{\varphi} N_1 + a^3 G_{\varphi\varphi} N_0 \varphi_1 + a^3 G_{\varphi\pi_{\varphi}} N_0 \pi_{\varphi 1} - 2a G_Y N_0 \nabla^2 \varphi_1 + \pi_0^{ij} \partial_i N_1^j. \quad (3.24)$$

Since the perturbed scalar field equation is linear in nature and follows wave equation, speed of sound is defined as the ratio of negative of the coefficient of  $\nabla^2 \varphi_1$  and  $\varphi_1''$  and in phase-space, it takes the form

$$c_s^2 = 2 N_0^2 a^4 G_{\pi_{\varphi} \pi_{\varphi}} G_Y \quad (3.25)$$

which, in conformal coordinate can be expressed as

$$c_s^2 = 2 a^6 G_{\pi_{\varphi} \pi_{\varphi}} G_Y. \quad (3.26)$$

The relation between generalized phase-space derivatives of  $G$  ( $G_{\varphi}$ ,  $G_Y$ ,  $G_{\varphi\pi_{\varphi}}$  etc.) and configuration-space derivatives of  $P(X, \varphi)$  ( $P$ ,  $P_{\varphi}$ ,  $P_{\varphi X}$  etc.) is unknown, hence, it is not possible to invert above Hamilton's equations to Euler-Lagrange equations and hence, it is not possible to compare both the formalisms. However, for a particular scalar field, the exact form of  $G$  is known to us (see Appendix A), and hence, for those model it is possible to write down equations of motion in configuration space and can be verified that Hamiltonian formulation of non-canonical scalar field is consistent.

#### 4 Inversion of non-canonical terms

In the last section, we showed that it is possible to obtain Hamiltonian for a non-canonical scalar field by defining a new variable  $G$  (see eqs. (3.5) and (3.8)). In order to understand the importance of this new function  $G$ , we ask the following question: *Starting from the Hamiltonian (3.10) and (3.18), can we invert the expressions leading to generalized equations in configuration-space?* In this section, we show that, inversion can be established.

To invert the equations, one needs to invert the coefficients like  $G_{\varphi}$ ,  $G_Y$ ,  $G_{\varphi\pi_{\varphi}}$  from phase-space to configuration-space. Since the form of  $G$  in configuration space is known, by carefully looking at the equations, it is apparent that only the phase-space derivatives of  $G$  are needed to invert which, in general, is not possible.

To begin with, let us take a phase-space function  $F \equiv F(\pi_{\varphi}, \gamma, Y, \varphi) = \tilde{F}(X, \gamma, Y, \varphi)$ , i.e.,

$$\begin{aligned} F &= F(\pi_{\varphi}, \gamma, Y, \varphi) \\ \Rightarrow dF &= F_{\pi_{\varphi}} d\pi_{\varphi} + F_{\gamma} d\gamma + F_Y dY + F_{\varphi} d\varphi \end{aligned}$$

Note that, tilde is used for configuration-space functions only. The invertibility of Legendre transformation implies that, if  $X = X(\pi_{\varphi}, \gamma, Y, \varphi)$  then  $\pi_{\varphi} = \pi_{\varphi}(\pi_{\varphi}, \gamma, Y, \varphi)$ , i.e.,

$$\begin{aligned} \pi_{\varphi} &= \pi_{\varphi}(X, \gamma, Y, \varphi) \\ d\pi_{\varphi} &= \frac{\partial \pi_{\varphi}}{\partial X} dX + \frac{\partial \pi_{\varphi}}{\partial \gamma} d\gamma + \frac{\partial \pi_{\varphi}}{\partial Y} dY + \frac{\partial \pi_{\varphi}}{\partial \varphi} d\varphi \end{aligned}$$

implying that

$$\begin{aligned} d\tilde{F}(X, \gamma, Y, \varphi) = & F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial X} dX + \left( F_\gamma + F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial \gamma} \right) d\gamma + \left( F_Y + F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial Y} \right) dY \\ & + \left( F_\varphi + F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial \varphi} \right) d\varphi. \end{aligned} \quad (4.1)$$

Hence, the relations between phase-space variables and configuration-space variables are

$$F_{\pi_\varphi} = \frac{\tilde{F}_X}{\frac{\partial \pi_\varphi}{\partial X}}, \quad (4.2)$$

$$F_\gamma = \tilde{F}_\gamma - F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial \gamma}, \quad (4.3)$$

$$F_Y = \tilde{F}_Y - F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial Y}, \quad (4.4)$$

$$F_\varphi = \tilde{F}_\varphi - F_{\pi_\varphi} \frac{\partial \pi_\varphi}{\partial \varphi}. \quad (4.5)$$

In our case, for arbitrary non-canonical scalar field, we do not know the exact form of  $G(\pi_\varphi, \gamma, Y, \varphi)$ , however, we know  $\tilde{G}(X, \gamma, Y, \varphi) = P - P_X(2X - Y)$ . Using equation (3.2) and the above established relations, we get

$$\begin{aligned} G_{\pi_\varphi} &= -\frac{\sqrt{-2X}}{a^3}, \quad G_{\pi_\varphi \pi_\varphi} = \frac{1}{a^6 (P_X + 2XP_{XX})}, \\ G_{\pi_\varphi \pi_\varphi \pi_\varphi} &= -\frac{\sqrt{-2X} (3P_{XX} + 2XP_{XXX})}{a^9 (P_X + 2XP_{XX})^3}, \\ G_\varphi &= P_\varphi, \quad G_{\varphi \pi_\varphi} = \frac{\sqrt{-2X} P_{X\varphi}}{a^3 (P_X + 2XP_{XX})}, \\ G_{\varphi \varphi} &= P_{\varphi\varphi} - \frac{2X P_{X\varphi}^2}{(P_X + 2XP_{XX})} \end{aligned} \quad (4.6)$$

Using these definitions it is possible to invert all phase-space quantities to the ones in configuration-space. To start with, we first consider speed of sound. In conformal coordinate, it is given by equation (3.26) and hence using above relations, we get

$$c_s^2 = \frac{P_X}{P_X + 2XP_{XX}} \quad (4.7)$$

which matches with the conventional configuration-space definition of sound of speed. Similarly, the zeroth order equations (4.8), (4.9) and (4.10), after inversion, become

$$H^2 = -\frac{\kappa}{3}(P_X \varphi_0'^2 + Pa^2), \quad \text{Hubble Constant: } H \equiv \frac{a'}{a} \quad (4.8)$$

$$-2\frac{a''}{a} + H^2 = \kappa Pa^2, \quad (4.9)$$

$$P_X \varphi_0'' - P_{XX} \varphi_0'' \varphi_0'^2 a^{-2} + P_{X\varphi} \varphi_0'^2 + 2P_X \varphi_0' H + P_{XX} H \varphi_0'^3 a^{-2} + P_\varphi a^2 = 0, \quad (4.10)$$

respectively. At first order,  $N_1 = a\phi_1$  and  $N^i = \delta^{ij}\partial_j B_1$ , which helps to reduce first order perturbed Hamiltonian constraint (3.22) into

$$\begin{aligned} \frac{H}{\kappa}\nabla^2 B_1 + \frac{3H^2}{\kappa}\phi_1 + \frac{G_{\pi\varphi}^2}{2G_{\pi\varphi\pi\varphi}}\phi_1 a^2 + \frac{G_{\pi\varphi}}{2G_{\pi\varphi\pi\varphi}a^2}\varphi_1' + \frac{G_{\pi\varphi}G_{\varphi\pi\varphi}}{2G_{\pi\varphi\pi\varphi}}\varphi_1 a^2 \\ - \frac{G_{\varphi}}{2}\varphi_1 a^2 = 0 \end{aligned} \quad (4.11)$$

which, further, can be inverted back to configuration-space, again by using above relations as

$$\begin{aligned} \mathcal{H}\nabla^2 B_1 = \frac{\kappa}{2}\left[P_X\phi_1\varphi_0'^2 + 2Pa^2\phi_1 + P_X\varphi_0'\varphi_1' + P_{XX}\phi_1\varphi_0'^4 a^{-2} - P_{XX}\varphi_1'\varphi_0'^3 a^{-2} + \right. \\ \left. P_{X\varphi}\varphi_0'^2\varphi_1 + P_{\varphi}\varphi_1 a^2\right]. \end{aligned} \quad (4.12)$$

Similarly, first order Momentum constraint becomes

$$\partial_i\phi_1 = -\frac{\kappa}{2H}P_X\varphi_0'\partial_i\varphi_1 \quad (4.13)$$

and equation of motion of scalar field  $\varphi_1$ , i.e., equation (3.24) takes the form

$$\begin{aligned} -P_X\varphi_1''a^2 - P_{XX}\phi_1'\varphi_0'^3 + P_{XX}\varphi_1''\varphi_0'^2 - P_{XX\varphi}\phi_1\varphi_0'^4 + P_{XX\varphi}\varphi_1'\varphi_0'^3 - P_{\varphi}\phi_1 a^4 - \\ P_{\varphi\varphi}a^4\varphi_1 + P_X\phi_1\varphi_0''a^2 + P_X\nabla^2\varphi_1 a^2 + P_X\phi_1'\varphi_0'a^2 - 2P_X\varphi_1'Ha^2 - 4P_{XX}\phi_1\varphi_0''\varphi_0'^2 + \\ 3P_{XX}\varphi_0'\varphi_1'' + P_{XX\varphi}\varphi_0''\varphi_0'^2\varphi_1 - P_{X\varphi}\varphi_0'\varphi_1'a^2 - P_{X\varphi}\varphi_0''a^2\varphi_1 - P_{X\varphi\varphi}\varphi_0'^2a^2\varphi_1 + \\ 2P_X\phi_1\varphi_0'Ha^2 + P_X\varphi_0'\nabla^2 B_1 a^2 + P_{XX}\phi_1H\varphi_0'^3 - P_{XX}\varphi_1'H\varphi_0'^2 - P_{XXX}\phi_1H\varphi_0'^5 a^{-2} + \\ P_{XXX}\phi_1\varphi_0''\varphi_0'^4 a^{-2} + P_{XXX}\varphi_1'H\varphi_0'^4 a^{-2} - P_{XXX}\varphi_1'\varphi_0''\varphi_0'^3 a^{-2} - P_{XX\varphi}H\varphi_0'^3\varphi_1 - \\ 2P_{X\varphi}\varphi_0'H\varphi_1 a^2 = 0. \end{aligned} \quad (4.14)$$

Equations (4.8), (4.9), (4.10), (4.12), (4.13) and (4.14) are consistent with the zeroth and first order perturbed Euler-Lagrange equations of motion.

## 5 Interaction Hamiltonian

The higher-order physical observables like Bi-spectrum/Tri-spectrum are related to higher-order correlation functions; in order to compute higher-order correlation functions, we need higher-order interaction Hamiltonian. In this section, we obtain the interaction Hamiltonian of the non-canonical field. Third order perturbed generalized interaction Hamiltonian for non-canonical scalar field in terms of phase-space variables is obtained directly by expanding the Hamiltonian (3.8) upto third order of perturbation [1] and it takes the form

$$\begin{aligned} \mathcal{H}_3 = -N_1\delta_{ij}\delta_{kl}\kappa\pi_1^{ij}\pi_1^{kl}a + 2N_1\delta_{ij}\delta_{kl}\kappa\pi_1^{ik}\pi_1^{jl}a - \frac{1}{2}G_{\varphi\varphi}N_1\varphi_1^2a^3 - \frac{1}{2}G_{\pi\varphi\pi\varphi}N_1\pi_{\varphi_1}^2a^3 - \\ G_{\varphi\pi\varphi}N_1\pi_{\varphi_1}a^3\varphi_1 - G_YN_1\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a - G_{Y\pi\varphi}N_0\pi_{\varphi_1}\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a - \\ G_{\varphi Y}N_0\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1\varphi_1a - \frac{1}{6}G_{\pi\varphi\pi\varphi\pi\varphi}N_0\pi_{\varphi_1}^3a^3 - \frac{1}{6}G_{\varphi\varphi\varphi}N_0\varphi_1^3a^3 - \\ \frac{1}{2}G_{\varphi\pi\varphi\pi\varphi}N_0\pi_{\varphi_1}^2a^3\varphi_1 - \frac{1}{2}G_{\varphi\varphi\pi\varphi}N_0\pi_{\varphi_1}\varphi_1^2a^3 + N_1^i\pi_{\varphi_1}\partial_i\varphi_1 \end{aligned} \quad (5.1)$$

and similarly, fourth order interaction Hamiltonian takes the form

$$\begin{aligned}
\mathcal{H}_4 = & -\frac{1}{2} G_{YY} N_0 \delta^{ij} \delta^{kl} \partial_i \varphi_1 \partial_j \varphi_1 \partial_k \varphi_1 \partial_l \varphi_1 a^{-1} - G_{\pi_\varphi Y} N_1 \pi_{\varphi 1} \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a - \\
& G_{\varphi Y} N_1 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 \varphi_1 a - \frac{1}{6} G_{\pi_\varphi \pi_\varphi \pi_\varphi} N_1 \pi_{\varphi 1}^3 a^3 - \frac{1}{6} G_{\varphi \varphi \varphi} N_1 \varphi_1^3 a^3 - \\
& \frac{1}{2} G_{\varphi \pi_\varphi \pi_\varphi} N_1 \pi_{\varphi 1}^2 a^3 \varphi_1 - \frac{1}{2} G_{\pi_\varphi \pi_\varphi Y} N_0 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 \pi_{\varphi 1}^2 a - \frac{1}{2} G_{\varphi \varphi \pi_\varphi} N_1 \pi_{\varphi 1} \varphi_1^2 a^3 - \\
& \frac{1}{2} G_{\varphi \varphi Y} N_0 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 \varphi_1^2 a - G_{\varphi \pi_\varphi Y} N_0 \pi_{\varphi 1} \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 \varphi_1 a - \\
& \frac{1}{24} G_{\pi_\varphi \pi_\varphi \pi_\varphi \pi_\varphi} N_0 \pi_{\varphi 1}^4 a^3 - \frac{1}{24} G_{\varphi \varphi \varphi \varphi} N_0 \varphi_1^4 a^3 - \frac{1}{6} G_{\varphi \pi_\varphi \pi_\varphi \pi_\varphi} N_0 \pi_{\varphi 1}^3 a^3 \varphi_1 - \\
& \frac{1}{6} G_{\varphi \varphi \varphi \pi_\varphi} N_0 \pi_{\varphi 1} \varphi_1^3 a^3 - \frac{1}{4} G_{\varphi \varphi \pi_\varphi \pi_\varphi} N_0 \pi_{\varphi 1}^2 \varphi_1^2 a^3. \tag{5.2}
\end{aligned}$$

Again, using inversion formulae mentioned in above section, phase-space form of interaction Hamiltonian can be written in terms of configuration-space variables.

## 6 Extension to generalized higher-derivative models

As we have shown above, for an arbitrary non-canonical scalar field, it is possible to define a canonical conjugate momenta and the Hamiltonian. In this section, we extend our method to generalize higher-derivative models. First, we extend the analysis to G-Inflation[11, 12] model with generalized functions  $P(X, \varphi)$  and  $K(X, \varphi)$ .

Action for G-Inflation scalar field minimally coupled to gravity is given by

$$S_G = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + P(X, \varphi) + K(X, \varphi) \square \varphi \right]. \tag{6.1}$$

Directly obtaining the Hamiltonian for the above action is difficult since it contains second order derivatives of the scalar field. However, using the approach of Deffayet *et al.* [1, 26], action (6.1) can be re-written as

$$\begin{aligned}
S_G = & \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + P(X, \varphi) + K(X, \varphi) S \right] + \int d^4x \lambda (S - \square \varphi) \\
= & \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + P(X, \varphi) + K(X, \varphi) S \right] + \int d^4x [\lambda S + \lambda g^{\mu\nu} \Gamma_{\mu\nu}^\alpha \partial_\alpha \varphi + \\
& g^{\mu\nu} \partial_\mu \varphi \partial_\nu \lambda + \lambda \partial_\nu g^{\mu\nu} \partial_\mu \varphi]. \tag{6.2}
\end{aligned}$$

Linearizing the action costs two extra variables in configuration-space, thus four extra phase-space variables. We have discussed the issue in I [1] and proved that those variables are not dynamic in nature and thus, there are no extra degrees of freedom.

Since the action (6.2) is converted in terms of first derivatives of fields, it is now possible to define momenta in terms of time derivative of the fields. However, the action still contains two generalized configuration-space variables  $P(X, \varphi)$  and  $G(X, \varphi)$ . Hence, using the above approach for generalized non-canonical scalar field, a consistent perturbed Hamiltonian formalism for generalized Galilean scalar field can be established.

The approach can also be extended to any other higher-derivative models like Hordenski scalar field models, modified gravity models or an arbitrary higher-derivative theory. The above case is the quadratic and cubic parts of the Hordenski's scalar field model[10]. In case

of general Hordenski's scalar field model, action depends on  $R_{\mu\nu}$ ,  $\nabla_{\mu\nu}\varphi$ ,  $\partial_\mu\varphi$ ,  $g_{\mu\nu}$  and  $\varphi$ . The metric-part can be written in terms of extrinsic tensor,  $K_{ij}$  and 3-Ricci scalar,  ${}^{(3)}R$  with Lapse function,  $N$  and Shift vector,  $N^i$ . Since the action contains  $\nabla_{\mu\nu}\varphi$  instead of only  $\square\varphi$ , above method for linearizing action will not work. Instead, we have to linearize the action by adding

$$S_H + \int d^4x \lambda^{\mu\nu} (S_{\mu\nu} - \nabla_{\mu\nu}\varphi)$$

for general Hordenski's model [26]. Hordenski's full action contains four unknown functions,  $G_n(X, \varphi)$ ,  $n = 2 \cdots 5$ , hence, using the approach for non-canonical scalar field, we can also deal with Hamiltonian formulation for Hordenski's theory.

By using the same argument and method, it is possible to obtain Hamiltonian beyond Hordenski's model, i.e., for any higher-order derivative gravity models with arbitrary functions. To deal with arbitrary functions, using the above approach for generalized non-canonical scalar fields, we can define a new and unique phase-space variable(s) and write the corresponding canonical Hamiltonian of the system and inversion formulae can be used to invert from phase-space variable to configuration-space variable and vice-versa. Once the Hamiltonian is obtained, we can use the approach in I to get the consistent Hamiltonian formalism of cosmological perturbation at any perturbed order for the specific model.

Hamiltonian approach in I, is independent of how we construct the Hamiltonian and is readily applicable once we successfully write down a consistent Hamiltonian for a specific model. Hence, the Hamiltonian approach for higher derivative theory is not restricted only by the Deffayet's approach [26]. Recently, Langlois and Noui [27, 28] have also provided a simpler way to obtain Hamiltonian for higher derivative theory and the Hamiltonian approach for perturbation can also be extended to these models.

## 7 Conclusion and discussion

In this work, we have explicitly provided the Hamiltonian formulation of cosmological perturbation theory for generalized non-canonical scalar fields. The following procedure was adopted: first we provided the essential information regarding gauge-choices and related gauge-invariant quantities. Next, we performed Legendre transformation for the generalized non-canonical scalar fields and showed that, since  $(\varphi' \rightarrow \pi_\varphi)$  transformation is not possible, Hamiltonian for generalized non-canonical scalar fields cannot be obtained by using conventional method.

We introduced a new generalized phase-space variable  $G(\pi_\varphi, \gamma, Y, \varphi)$  that is unique for different non-canonical scalar fields and obtained Hamiltonian of a non-canonical scalar field. We showed that, this is the simplest and efficient way to obtain the Hamiltonian. We extended the approach in I to generalized non-canonical scalar fields in the flat-slicing that doesn't lead to gauge-artifacts and obtained perturbed Hamilton's equations in terms of phase-space variables. In parallel, we also extended Langlois' approach to generalized non-canonical scalar field and showed that both approaches lead to identical speed of sound.

In order to compare Hamiltonian approach with Lagrangian approach, Hamilton's equations are to be converted to Euler-Lagrange equation and in doing so, we provided explicit forms of  $G(\pi_\varphi, \gamma, Y, \varphi)$  for different non-canonical scalar field models and showed that the Hamiltonian formulation is consistent.

Since we do not know how, in general, phase-space derivatives of  $G(\pi_\varphi, \gamma, Y, \varphi)$  transform to configuration-space derivatives, hence for an arbitrary field, it is not possible to directly

invert the generalized phase-space Hamilton's equations to Euler-Lagrange equations. In order to overcome this, we prescribed an inversion mechanism from generalized phase-space variables to generalized configuration-space variables (and vice versa) and showed that all generalized phase-space equations lead to consistent E-L equations. We also retrieved the conventional form of speed of sound in configuration-space.

We also obtained the interaction Hamiltonian in terms of phase-space variables for generalized non-canonical scalar field at third and fourth order of perturbation for scalar perturbations. These can also be expressed in terms of  $(\varphi', \varphi)$  using the general inversion formulae. Note that, we considered only the first order scalar perturbations. Vector or tensor modes can similarly be implemented by considering  $\delta\gamma_{ij} \neq 0$  and decomposing the metric using vector and tensor modes. Hamiltonian as well as equations of motions for vector or tensor modes also change accordingly. For the linear order, three modes decouple and  $\delta\pi^{ij}$  can also be decomposed as  $\delta\pi_S^{ij} + \delta\pi_V^{ij} + \delta\pi_T^{ij}$ , so the equations of motion. However, for higher order of perturbations, modes are highly coupled to each other, hence similar decomposition is not possible.

Finally, we briefly discussed the Hamiltonian formulation for generalized higher derivative scalar fields. The method is not restricted to gravity related models, it can also be applied to any other models where the Lagrangian is not specified properly.

Throughout the work, we carried out the method by assuming that the field allows Legendre transformation, which, most of the known models follow. However, if a certain model is specified in such a way that  $\varphi'$  cannot be written in terms of  $\pi_\varphi$  or the mapping is one-to-many, then the current formalism cannot be applied to obtain a unique form of  $G(\pi_\varphi, \gamma, Y, \varphi)$  and hence, for those kind of models, this approach is not applicable.

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## A Inversion formulae of $X$ and $\pi_\varphi$ and $G(\gamma, \pi_\varphi, Y, \varphi)$ for different scalar field models

### A.1 Canonical scalar field

In case of canonical scalar field,  $P(X, \varphi)$  is given by

$$P(X, \varphi) = -X - V(\varphi).$$

Hence, using equation (3.2), we get

$$\begin{aligned} \pi_\varphi &= \sqrt{\gamma} \sqrt{-2X + Y} \\ \Rightarrow X &= \frac{1}{2}Y - \frac{\pi_\varphi^2}{2\gamma} \end{aligned} \tag{A.1}$$

and  $G(\gamma, \pi_\varphi, Y, \varphi)$  is given by the relation (3.5)

$$G(\gamma, \pi_\varphi, Y, \varphi) = -\frac{1}{2} \frac{\pi_\varphi^2}{\gamma} - \frac{1}{2} Y - V(\varphi). \tag{A.2}$$

### A.2 Tachyonic field

Tachyons are described by

$$P(X, \varphi) = -V(\varphi)\sqrt{1 + 2X}.$$

Similarly, in case of Tachyons, we get

$$X = \frac{\gamma V^2 Y - \pi_\varphi^2}{2(\pi_\varphi^2 + \gamma V^2)} \quad (\text{A.3})$$

$$G(\gamma, \pi_\varphi, Y, \varphi) = -\frac{1}{\sqrt{\gamma}} \sqrt{1 + Y} \sqrt{\pi_\varphi^2 + \gamma V^2}. \quad (\text{A.4})$$

### A.3 DBI field

For DBI field,

$$P(X, \varphi) = -\frac{1}{f(\varphi)} \left( \sqrt{1 + 2f(\varphi)X} - 1 \right) - V(\varphi)$$

which implies that,

$$X = \frac{\gamma Y - \pi_\varphi^2}{2(\gamma + f \pi_\varphi^2)} \quad (\text{A.5})$$

$$G(\gamma, \pi_\varphi, Y, \varphi) = -\frac{1}{\gamma f(\varphi)} \sqrt{(\gamma + f(\varphi)Y)(f(\varphi)\pi_\varphi^2 + \gamma)} + \frac{1}{f(\varphi)} - V(\varphi) \quad (\text{A.6})$$

## B Langlois' approach for non-canonical scalar field

Two decades back, Langlois' obtained a consistent Hamiltonian for canonical scalar field[24]. In this section, we extend the method to non-canonical scalar fields.

Following [24], expressing background 3-metric  $\gamma_{0ij} = e^{2\alpha}$ , it can be shown that the first order perturbed Hamiltonian constraint takes the form

$$\begin{aligned} \mathcal{H}_{N1} \equiv & -\frac{e^{3\alpha}}{2\kappa} \left[ \gamma_0^{ik} \gamma_0^{jl} - \gamma_0^{ij} \gamma_0^{kl} \right] \partial_{ij} \gamma_{1kl} - \frac{\kappa}{3} e^{-3\alpha} \pi_\alpha \gamma_{0ij} \pi_1^{ij} - \left[ \frac{\kappa}{72} e^{-3\alpha} \pi_\alpha^2 + \right. \\ & \left. \frac{1}{2} e^{3\alpha} G \right] \gamma_0^{ij} \gamma_{1ij} - e^{3\alpha} G_{\pi_\varphi} \pi_{\varphi 1} - e^{3\alpha} G_\varphi \varphi_1 = 0, \end{aligned} \quad (\text{B.1})$$

where  $\pi_\alpha$  is the momentum corresponding to  $\alpha$ . Similarly, first order perturbed Momentum constraint becomes

$$\mathcal{H}_{1i} \equiv -2 \partial_k \gamma_{1ij} \pi_0^{jk} - 2 \gamma_{0ij} \partial_k \pi_1^{jk} + \pi_0^{jk} \partial_i \gamma_{1jk} + \pi_{\varphi 1} \partial_i \varphi_1 = 0. \quad (\text{B.2})$$

In momentum space, equations (B.1) and (B.2) becomes

$$\begin{aligned} \mathcal{H}_{N1}(k) \equiv & -\frac{e^{3\alpha}}{2\kappa} \left[ \gamma_0^{ik} \gamma_0^{jl} - \gamma_0^{ij} \gamma_0^{kl} \right] k_i k_j \gamma_{1kl} - \frac{\kappa}{3} e^{-3\alpha} \pi_\alpha \gamma_{0ij} \pi_1^{ij} - \left[ \frac{\kappa}{72} e^{-3\alpha} \pi_\alpha^2 + \right. \\ & \left. \frac{1}{2} e^{3\alpha} G \right] \gamma_0^{ij} \gamma_{1ij} - e^{3\alpha} G_{\pi_\varphi} \pi_{\varphi 1} - e^{3\alpha} G_\varphi \varphi_1 = 0 \end{aligned} \quad (\text{B.3})$$

$$\mathcal{H}_{1i}(k) \equiv -2 k_k \gamma_{1ij} \pi_0^{jk} - 2 \gamma_{0ij} k_k \pi_1^{jk} + \pi_0^{jk} k_i \gamma_{1jk} + \pi_{\varphi 1} k_i \varphi_1 = 0 \quad (\text{B.4})$$



The scalar configuration variables are

$$\gamma_1 = \frac{1}{3}\gamma_0^{ij} \gamma_{1ij}, \quad \gamma_2 = \frac{1}{2} \left[ \frac{3k^i k^j}{k^2} \gamma_{1ij} - \gamma_0^{ij} \gamma_{1ij} \right], \quad (\text{B.5})$$

which are associated with their conjugate momenta

$$\pi^1 = \gamma_{0ij} \pi_1^{ij}, \quad \pi^2 = \frac{k_i k_j}{k^2} \pi_1^{ij} - \frac{1}{3} \gamma_{0ij} \pi_1^{ij}. \quad (\text{B.6})$$

Hence the energy constraint (B.3) becomes

$$E = - \left[ \frac{1}{24} \kappa e^{-3\alpha} \pi_\alpha^2 + \frac{3}{2} e^{3\alpha} G \right] \gamma_1 - \frac{e^{3\alpha}}{\kappa} k^2 \gamma_1 + \frac{e^{3\alpha}}{3\kappa} k^2 \gamma_2 \\ - \frac{\kappa}{3} e^{-3\alpha} \pi_\alpha \pi^1 - e^{3\alpha} G_{\pi_\varphi} \pi_{\varphi 1} - e^{3\alpha} G_\varphi \varphi_1 = 0. \quad (\text{B.7})$$

Momentum constraint contains scalar and vector, both modes. Contracting with  $k^i$ , we obtain the scalar part of Momentum constraint

$$M \equiv \frac{1}{6} \pi_\alpha \gamma_1 - \frac{2}{9} \pi_\alpha \gamma_2 - \frac{2}{3} \pi^1 - 2\pi^2 + \pi_\alpha \varphi_1 = 0 \quad (\text{B.8})$$

In case of scalar phase-space, there exist two first class constraints, namely  $E$  and  $M$  and

$$E \left( \gamma_\alpha, \pi^\beta = \frac{\partial S}{\partial \gamma_\beta} \right) = 0 \quad (\text{B.9})$$

$$M \left( \gamma_\alpha, \pi^\beta = \frac{\partial S}{\partial \gamma_\beta} \right) = 0 \quad (\text{B.10})$$

where  $S$  is the quadratic generating function, given by

$$S = \frac{1}{2} A_{\alpha\beta} \gamma_\alpha \gamma_\beta + B_\alpha \gamma_\alpha. \quad (\text{B.11})$$

$\alpha, \beta = 0, 1, 2, \gamma_0 = \varphi_1, \pi^0 = \pi_{\varphi 1}$  and  $A_{\alpha\beta}$  are symmetric. Hence, equations (B.9) and (B.10) become equations for  $A_{\alpha\beta}$  and  $B_\alpha$  with a polynomial form in  $\gamma_\alpha$  and lead to the following four equations for the Energy constraint:

$$- \left[ \frac{1}{24} \kappa e^{-3\alpha} \pi_\alpha^2 + \frac{3}{2} e^{3\alpha} G \right] - \frac{e^{3\alpha}}{\kappa} k^2 - \frac{e^{-3\alpha} \kappa}{3} \pi_\alpha A_{11} - e^{3\alpha} G_{\pi_\varphi} A_{01} = 0 \quad (\text{B.12})$$

$$\frac{e^{3\alpha}}{3\kappa} k^2 - \frac{\kappa}{3} e^{-3\alpha} \pi_\varphi A_{12} - e^{3\alpha} G_{\pi_\varphi} A_{02} = 0 \quad (\text{B.13})$$

$$- \frac{\kappa}{3} e^{-3\alpha} \pi_\alpha A_{01} - e^{-3\alpha} G_{\pi_\varphi} A_{00} - e^{3\alpha} G_\varphi = 0 \quad (\text{B.14})$$

$$- \frac{\kappa}{3} e^{-3\alpha} \pi_\alpha B_1 - e^{3\alpha} G_{\pi_\varphi} B_0 = 0 \quad (\text{B.15})$$

and for the Momentum constraint

$$\frac{1}{6} \pi_\alpha - \frac{2}{3} A_{11} - 2A_{21} = 0 \quad (\text{B.16})$$

$$- \frac{2}{9} \pi_\alpha - \frac{2}{3} A_{12} - 2A_{22} = 0 \quad (\text{B.17})$$

$$- \frac{2}{3} A_{10} - 2A_{20} + \pi_\alpha = 0 \quad (\text{B.18})$$

$$- \frac{2}{3} B_1 - 2B_2 = 0 \quad (\text{B.19})$$

respectively.

The solutions for the  $B_\alpha$  form a one-dimensional space and can be written as

$$B_0 = P, \quad B_1 = -\frac{3}{\kappa} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} B_0, \quad B_2 = \frac{e^{6\alpha}}{\kappa} \frac{G_{\pi_\varphi}}{\pi_\alpha} B_0 \quad (\text{B.20})$$

where dependence of the  $B_\alpha$  on the free parameter  $P$  is chosen for later convenience.  $A_{\alpha\beta}$  are undetermined since there are five out of six independent equations and one is background equation. The additional condition is arbitrary and independent of any physical change in the system. The quantity  $P$  is the momentum in the reduced phase-space and its conjugate coordinate is given by

$$Q = \frac{\partial S}{\partial P} = \varphi_1 + \frac{e^{6\alpha}}{\kappa} \frac{G_{\pi_\varphi}}{\pi_\alpha} (\gamma_2 - 3\gamma_1) \quad (\text{B.21})$$

which coincides with gauge-invariant Mukhanov's variable. Other relations between old and new variables are given as

$$\varphi_1 = Q + [\gamma_1, \gamma_2], \quad \pi_{\varphi 1} = A_{00} Q + P + [\gamma_1, \gamma_2] \quad (\text{B.22})$$

$$\pi^1 = A_{10} Q - \frac{3}{\kappa} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} P + [\gamma_1, \gamma_2], \quad \pi^2 = A_{20} Q + \frac{e^{6\alpha}}{\kappa} \frac{G_{\pi_\varphi}}{\pi_\alpha} P + [\gamma_1, \gamma_2] \quad (\text{B.23})$$

where brackets contain all the terms with  $\gamma_1$  or  $\gamma_2$ . These are not written explicitly since they are 'pure gauge' and do not contribute to the 'true' dynamics.

The second order expansion of the Energy constraint is given by

$$\begin{aligned} \mathcal{H}_{N2} = & \frac{2\kappa}{\sqrt{\gamma}} \left( \gamma_{0ik} \gamma_{0jl} - \frac{1}{2} \gamma_{0ij} \gamma_{0kl} \right) \pi_1^{ij} \pi_1^{kl} - \frac{\sqrt{\gamma}}{2} G_{\pi_\varphi \pi_\varphi} \pi_{\varphi 1}^2 \\ & - \frac{\sqrt{\gamma}}{2} G_{\varphi\varphi} \varphi_1^2 - \sqrt{\gamma} G_Y \gamma_0^{ij} \partial_i \varphi_1 \partial_j \varphi_1 - \sqrt{\gamma} G_{\varphi\pi_\varphi} \pi_{\varphi 1} \varphi_1 + [\gamma_{ij}] \end{aligned} \quad (\text{B.24})$$

where  $[\gamma_{ij}]$  collectively represents all the terms that involve  $\gamma_{1ij}$ . By choosing  $N = 1$  and  $N^i = 0$  to simplify calculations, the scalar part of the Hamiltonian is easily obtained and is given by

$$\begin{aligned} H^s = & \int d^3k \{ N \mathcal{H}_N + N^i \mathcal{H}_i \} \\ = & \int d^3k \left\{ \frac{2\kappa}{\sqrt{\gamma}} \left( -\frac{1}{6} \pi_1^2 - \frac{3}{2} \pi_2^2 \right) - \frac{\sqrt{\gamma}}{2} G_{\pi_\varphi \pi_\varphi} \pi_{\varphi 1}^2 \right. \\ & \left. - \frac{\sqrt{\gamma}}{2} G_{\varphi\varphi} \varphi_1^2 - \sqrt{\gamma} G_Y k^2 \varphi_1^2 - \sqrt{\gamma} G_{\varphi\pi_\varphi} \pi_{\varphi 1} \varphi_1 + [\gamma_1, \gamma_2] \right\}. \end{aligned} \quad (\text{B.25})$$

Hence the gauge-invariant Hamiltonian is given by

$$H_{GI}^s = H^s + \{S, H_0\}_{\text{Background}} \quad (\text{B.26})$$

$$\begin{aligned} = & \int d^3k \left[ -\frac{\sqrt{\gamma}}{2} G_{\pi_\varphi \pi_\varphi} P^2 + \left\{ -\frac{\kappa}{3\sqrt{\gamma}} A_{10}^2 + \frac{3\kappa}{\sqrt{\gamma}} A_{20}^2 - \frac{\sqrt{\gamma}}{2} G_{\pi_\varphi \pi_\varphi} A_{00}^2 \right. \right. \\ & \left. - \frac{\sqrt{\gamma}}{2} G_{\varphi\varphi} + \sqrt{\gamma} G_Y k^2 + \frac{1}{2} \dot{A}_{00} - \sqrt{\gamma} G_{\varphi\pi_\varphi} A_{00} \right\} Q^2 \\ & \left. + \left\{ \frac{2}{\sqrt{\gamma}} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} A_{10} + \frac{6}{\sqrt{\gamma}} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} A_{20} - \sqrt{\gamma} G_{\pi_\varphi \pi_\varphi} A_{00} \right\} P Q \right] \end{aligned} \quad (\text{B.27})$$

where  $\dot{A}_{00} = \{A_{00}, H_0\}_{Background}$ . If we impose additional condition

$$\frac{2}{\sqrt{\gamma}} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} A_{10} + \frac{6}{\sqrt{\gamma}} e^{6\alpha} \frac{G_{\pi_\varphi}}{\pi_\alpha} A_{20} - \sqrt{\gamma} G_{\pi_\varphi \pi_\varphi} A_{00} = 0 \quad (\text{B.28})$$

in order to cancel cross terms in the above Hamiltonian, we get the following solutions:

$$\begin{aligned} A_{00} &= \frac{3 G_{\pi_\varphi}}{G_{\pi_\varphi \pi_\varphi}} \frac{\pi_{\varphi 0}}{\pi_\alpha}, \quad A_{10} = -\frac{9}{\kappa} e^{6\alpha} \frac{G_{\pi_\varphi}^2 \pi_{\varphi 0}}{G_{\pi_\varphi \pi_\varphi} \pi_\alpha^2} - \frac{3}{\kappa} e^{6\alpha} \frac{\pi_{\varphi 0}}{\pi_\alpha}, \\ A_{20} &= \frac{3}{\kappa} e^{6\alpha} \frac{G_{\pi_\varphi}^2 \pi_{\varphi 0}}{G_{\pi_\varphi \pi_\varphi} \pi_\alpha^2} + \frac{1}{\kappa} e^{6\alpha} \frac{\pi_{\varphi 0}}{\pi_\alpha} + \frac{1}{2} \pi_{\varphi 0}. \end{aligned} \quad (\text{B.29})$$

Finally, the Hamiltonian takes the form

$$H_{GI}^s = \int d^3k \left\{ -\frac{1}{2} e^{3\alpha} G_{\pi_\varphi \pi_\varphi} P^2 + \frac{1}{2} (X + 2 e^{3\alpha} k^2 G_Y) Q^2 \right\}, \quad \text{where} \quad (\text{B.30})$$

$$\begin{aligned} X \equiv 9 e^{3\alpha} \frac{G_{\pi_\varphi}^2 \pi_{\varphi 0}^2}{G_{\pi_\varphi \pi_\varphi} \pi_\alpha^2} - 6 e^{3\alpha} \frac{G_{\varphi \pi_\varphi} G_{\pi_\varphi} \pi_{\varphi 0}}{\pi_\alpha} + \dot{A}_{00} + \frac{3}{2} \kappa e^{-3\alpha} \pi_{\varphi 0}^2 \\ 6 e^{3\alpha} \frac{G_\varphi \pi_{\varphi 0}}{\pi_\alpha} - e^{3\alpha} G_{\varphi \varphi} \end{aligned} \quad (\text{B.31})$$

and the corresponding equation of motion becomes

$$\ddot{Q} - 2 k^2 e^{6\alpha} G_{\pi_\varphi \pi_\varphi} G_Y Q + \left( 3 H - \frac{\dot{G}_{\pi_\varphi \pi_\varphi}}{G_{\pi_\varphi \pi_\varphi}} \right) \dot{Q} - e^{3\alpha} G_{\pi_\varphi \pi_\varphi} X Q = 0. \quad (\text{B.32})$$

Note that, speed of sound

$$\begin{aligned} c_s^2 &= 2 e^{6\alpha} G_{\pi_\varphi \pi_\varphi} G_Y \\ &= 2 a^6 G_{\pi_\varphi \pi_\varphi} G_Y. \end{aligned} \quad (\text{B.33})$$

Note that, in Langlois' approach, the time coordinate represents cosmic time and hence the sound speed, according to our approach, is given by  $2 a^4 G_{\pi_\varphi \pi_\varphi} G_Y$ . The discrepancy arises due to the fact that, in Langlois' approach,  $\gamma^{ij} \partial_{ij} \rightarrow -k^2$  (see eqs. (B.24) and (B.25)), where we have used  $\delta^{ij} \partial_{ij} \rightarrow -k^2$ . Hence, extra  $a^2$  factor appears in Langlois' approach.

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